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Review

Behavioral finance in financial market theory, utility theory, portfolio theory and the necessary statistics: A review

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ABSTRACT

We present an overview of behavioral finance's consistent role in portfolio theory and market theory through utility theory. Since Bernoulli, the subjective nature of utility has been increasingly generalized for questionable purposes. Behavioral finance is reverting back to the original intents of utility theory. We also examine the statistical methods used to determine their suitability for the task at hand. Given the heterogeneous population at the market and individual security level, we suggest that nonparametric nonlinear statistics are best suited for descriptive and inferential analysis of all possible investor preferences.

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1. Introduction

The major challenge facing behavioral finance is to evolve toward an integrated theory of financial market operations. This challenge has been issued by traditional finance theorists many times. Fama (2012) states that while behaviorists are very good at "story telling" and describing individual behavior, their jumps from individuals to markets are not validated by the data. The purpose of this paper is to suggest a path toward an integrated behavioral finance theory using utility theory and portfolio theory. Portfolio theory is important because behavioral theory

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tends to focus on individual behavior or psychology instead of group or organizational behavior with a focus on social psychology. Portfolio theory specifically concentrates on the nonlinear interrelationships between micro-units in order to build an integrated portfolio. Portfolios simply are not a linear sum of the parts. Instead of the traditional mean–variance portfolio theory, we propose the use of UPM/LPM portfolio theory based on partial moments which provides the benefits of nonparametric statistics and expected utility theory.

One issue is that the traditional approach uses mathematics to build financial theories. Unfortunately, mathematical models require boundary conditions (assumptions) in order to generate a closed form solution. The devil is in the assumptions—primarily the rational investors, symmetric information and no market cost assumptions. With those assumptions, we are able to generate beautiful closed form market models. Without those assumptions, we lose some of the simple beauty of mathematics but hopefully are able to derive a better understanding of markets. We still can use mathematics and statistics on closed form micro-models while making fewer assumptions. But in the end, we have to give up the vision of a mathematical theory of everything promised by the traditional approach.

This paper consists of a review of the relevant literature in market theory, utility theory, and portfolio theory. We hope to be able to provide a viewpoint that allows the integration of the three while achieving the benefits resulting from the study of behavioral finance.

2. Towards an integrated financial theory

An integrated financial theory requires a market theory, an economic utility theory, and a portfolio selection model. First let us look at the market theory. If a market is perfectly efficient with a Walras equilibrium for every Pareto optimum transaction in the market, we should have a stationary probability distribution, either normal or a Mandelbrot stable paretian. This type of a market is very easy to model mathematically and can easily integrate micro- and macro-behavior. Wiener (1948) was one of the first researchers to reject the rational investor assumption inherent in this efficient market theory. He asserted that rational investors would resort to lying, cheating and stealing in order to maximize their utility and society would react by placing them in prison. He also stated that financial institutions would not exist if everyone was rational, because a generic institutional portfolio would not be able to maximize utility for every member of the institution. Rational investors do not play well in a group. As a result, institutional economics theory will not include rational participants and for the most point this is true. If we follow the institutional theories of Coase (1937), March and Simon (1958), Cyert and March (1963) and Williamson (2002),[1] we see the concept of transaction costs and bounded rationality allowing organizations to exist within the financial markets alongside with a rejection of efficient market theory.

Now it is not binary so we do not have a choice between an efficient market and an inefficient market. There is a wide gulf in between. The area that is between inefficient and efficient markets is an effective market. Effective markets are quite complex and basically do the job as we do not have a better alternative (Marxism anyone?). Effective markets are the result of transaction costs, asymmetric information and bounded rationality resulting in dynamic homeostasis systems following the second law of thermodynamics that are going to not only generate non-normal distributions but also non-stationary distributions, i.e., the moments of the distributions are going to change over time.

CAPM, APT, or any general asset pricing models are classical (static) equilibrium models that have to rely on unrealistic assumptions in order to provide boundary conditions for a mathematical solution. The major assumption is one of linear or risk-neutral utilities. Unfortunately, Roll (1977) found that the mathematical model in the case of CAPM is not empirically testable. Not surprisingly, we do not have any empirical support for CAPM.[2] CAPM derives from Markowitz's modern portfolio theory (MPT) but adds a number of unrealistic assumptions to provide the boundary conditions for a closed-form mathematical solution. MPT does not make these assumptions. Thus MPT is more realistic but the result is a model that is limited to a smaller micro- state in order to maintain a closed-form solution. It does not provide us with a macroeconomic model of asset pricing in our capital markets. Thus, asset pricing and MPT are two different things. We do not use MPT as an asset pricing model because we have not made the assumptions to make the asset pricing model a closed form solution. Markowitz (2010) is on record as not supporting CAPM because of the unrealistic assumptions required for CAPM but not required for his MPT. The assumptions include: lending and borrowing at the risk free rate of return, unlimited borrowing, short-selling without margin requirements, homogeneous expectations and risk-neutral utility theory. When these assumptions are eliminated, the capital market line becomes nonlinear and requires utility theory in order to maximize investor utility. One negative result of the popularity of the CAPM is the elimination of expected utility theory and utility theory is at the heart of Markowitz's MPT.

Theories of markets operating in non-stationary disequilibria have been around for quite a while in the institutional/evolutionary/energy economics area of economic thought. First, we have the Coase–Simon–March–Cyert–Williamson behavioral theory in institutional economics. We also have the Fractal/Chaos theory model developed in the 1960s through 1980s by Mandelbrot (Mandelbrot and Hudson, 2004) and popularized by Peters (1991, 1994), the evolutionary theory of Georgescu-Roegen (1971) and Boulding (1981, 1991), the bifurcation market theory of

[1] We would be remiss if we failed to note that Coase, Simon, and Williamson are Nobel Laureates in economics because of their work in this area. Cyert and March (1963) wrote one of the first behavioral finance books.

[2] "Low vol" strategies are currently demonstrating the inverse relationship between risk and reward as postulated in CAPM.

Nawrocki (1984, 1995), and the adaptive markets hypothesis of Lo (2004). Nawrocki's (1984, 1995) bifurcation market model[3] was based on Wiener (1948) and Shannon and Weaver's (1949) work on information theory, Murphy's (1965) work in adaptive control processes in economic systems, Georgescu-Roegen's (1971) entropy model of economic theory, Nicolis and Prigogine's dissipative structures (1977),[4] and Boulding's (1981, 1991) evolutionary economics. This work resulted in Nawrocki and Vaga's (2013) bifurcation parameter for understanding dynamic market disequilibria. While the motivating forces for these dynamic market models are economic expectations and behavioral institutional theories, the resulting quantitative theories use the mathematical and statistical tools developed by Wiener, Shannon and Weaver, Mandelbrot, Murphy, Nicolis and Prigogine, and Nawrocki and Vaga.

It would be fair at this point to question the benefit of moving to these dynamic disequilibria models. While these models are unlikely to lead to a comprehensive closed-form mathematical model of market operation, we can study micro- areas of the market quantitatively using very few boundary conditions, i.e., unrealistic assumptions. The major advantage to these disequilibria models is that the random walk market model is a subset of the dynamic disequilibria model. Indeed, we can provide a proof for the random walk model using the information (entropy) theory.[5] *To repeat, the random walk/efficient market model is a subset of the dynamic disequilibria model.* However, the dynamic disequilibria model is not a subset of the random walk/efficient market model. In practice, this means that there are no anomalies to be discovered with the dynamic disequilibria model. The number of discovered anomalies to the random walk/efficient market model numbers in the thousands. Which methodology would you prefer? (1) A methodology that has the ability to discover an equilibrium result as well as a non-equilibrium result or (2) a methodology that can only discover an equilibrium result with every other result considered an anomaly.

3. Utility theory

With the rejection of static asset pricing models, we need to rediscover economic utility theory. Markowitz (1959) spends about a quarter of his book describing utility theory. Very simply, you cannot do MPT analysis without utility theory. Markowitz's quadratic utility function used in mean–variance analysis always had a slope coefficient that captures investor risk-return tradeoffs so there were always numerous optimal portfolios on the efficient frontier depending on the specific slope coefficient. Markowitz

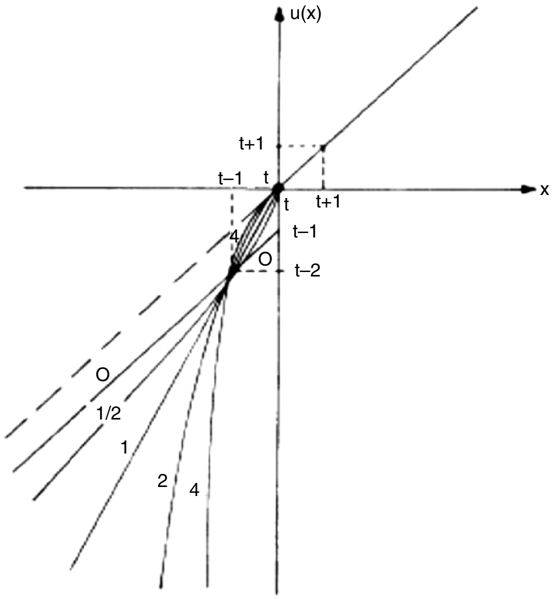

Fig. 1. Plots of LPM utility functions for five values of *a*.
Source: Fishburn (1977).

is aware of the numerous paradoxes and puzzles associated with utility theory and he is aware of non-normal distributions so he has always argued that the quadratic utility function is a reasonable approximation of a rational investor. However, there are other behavioral aspects beyond risk-aversion that create these puzzles and paradoxes[6] but still leave us with a final answer of bounded rational behavior. The use of utility theory moves us to mean-LPM (Bawa, 1975; Fishburn, 1977) as shown in Fig. 1 and UPM/LPM portfolio selection models (Fishburn and Kochenberger, 1979; Holthausen, 1981) as shown in Fig. 2 because they provide a richer, more realistic utility theory than mean–variance and are bounded rationality models as developed in March and Simon (1958). But if you are a believer in an asset pricing model, then you have no interest in utility theory because it has been assumed away.

Utility theory starts with Bernoulli (1954). Giocoli (1998) revisits the original Bernoulli text in order to preserve the intended message from utility theory.

> In his 1738 essay Daniel Bernoulli suggests a new criterion to determine the fair price. His problem, therefore, is the same as Huygens's. The novelty of Bernoulli is in the answer, not in the question.
>
> According to Bernoulli, the expected value rule is based upon an implicit hypothesis: the independence of the value of the game from the evaluator (see footnote 8). This means that the price of the bet is an objective 'quantity', which can be determined by a super partes judge. No subjective characteristic of the players is relevant.
>
> The breaking with the tradition of Huygens, Montmort and de Moivre is here. For Bernoulli, it is not

3 Bifurcation theory processes are adaptive processes.

4 Ilya Prigogine is best known for his definition of dissipative structures and their role in thermodynamic systems far from equilibrium, a discovery that won him the Nobel Prize in Chemistry in 1977. In summary, Ilya Prigogine discovered that importation and dissipation of energy into chemical systems could reverse the maximization of entropy rule imposed by the second law of thermodynamics. Dissipative structure theory led to pioneering research in self-organizing systems which serves as a bridge between natural sciences and social sciences and between general systems theory and thermodynamics.

5 Cozzolino and Zahner (1973).

6 See Ellsberg (1961) for a treatment on ambiguity aversion and rational behavior.

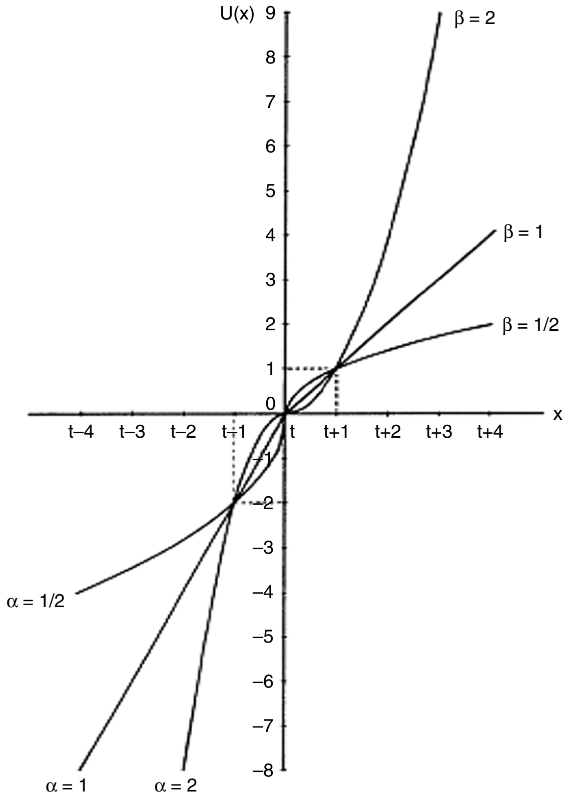

Fig. 2. Plots of the UPM/LPM utility function for $k = 2$ and various α, β values.
Source: Holthausen (1981).

possible to solve the problem of the value of a game according to an objective evaluation, because 'value' is always a subjective judgement in which a decisive role is played by the evaluator's individual characteristics and circumstances. Therefore, the criterion to determine the price of a bet must take into account also the player, and not only the 'rules of the game'.

The importance of Bernoulli's contribution to the theory of decision under uncertainty is manifest. If the decision problem could be solved only upon objective features, its solution would always be the same and the whole issue would be trivial. *The problem is interesting, on the contrary, precisely because its solution depends on the decision maker's behaviour according to his subjective preferences over the uncertain outcomes*.

The real novelty of Bernoulli is to have centred the choice problem on the subject, and on the object, of the decision (see footnote 9). All the rest of the 1738 essay is simply an extension of this thesis, in particular, an effort of making it 'operational' in order to stand the competition of the Huygens's approach (see next section).

Bachelier (1900) affirms the personal nature of probabilities and therefore, utility:

> With probabilities in the operations of the Stock Exchange, two kinds of probabilities can be considered:
> (1) Probability that might be called mathematical; this is that which can be determined a priori; that which is studied in games of chance.
> (2) Probability depending on future events and, as a consequence, impossible to predict in a mathematical way.

It is the latter probability that a speculator seeks to predict. He analyses the reasons which may influence rises or falls in prices and the amplitude of price movements. His conclusions are completely personal as his counter-party necessarily has the opposite opinion.

Bernoulli proposes the first, *and more important*, of his hypotheses:

> The value of a good is not given by its price, but by its utility.
>
> The utility of a good is subjective. It depends upon the individual circumstances of the decision-maker. This explains why a lottery ticket may be valued differently according to the wealth of the individual.
>
> Here lies the core of Bernoulli's memoir: the fair price of a game must be determined starting from the assumption that value is a subjective concept. This is the 'true' hypothesis of Bernoulli. Giocoli (1998).

Modern utility theory starts with Von Neumann and Morgenstern (1947). Friedman and Savage (1948) and Markowitz (1952) provide reverse S-shaped utility functions. In the late 1970s, the S-shaped utility functions of Kahneman and Tversky's (1979) prospect theory were introduced. Bawa (1975) and Fishburn (1977) provided proofs that mean-lower partial moment models could implement von Neumann and Morgenstern utility functions. Fishburn and Kochenberger (1979) and Holthausen (1981) introduced UPM-LPM models that can implement reverse S-shaped utility functions of Friedman, Savage and Markowitz and the S-shaped utility functions of Kahneman and Tversky in addition to Von Neumann and Morgenstern. Referring back to Fig. 2, the $\alpha = 2$, $\beta = 2$ utility line[7] is the reverse S-shaped utility function and the $\alpha = 1/2$, $\beta = 1/2$ utility line is the S-shaped utility function. Very simply, the UPM-LPM utility model can be fitted to any individual utility function known in the literature as demonstrated by Viole and Nawrocki (2011, 2013).

4. Portfolio theory

There are two portfolio theories that developed in parallel. They are (1) Markowitz's (1959) Bayesian-based MPT and (2) Normative Portfolio Theory (NPT) by Frankfurter and Phillips (1995). Frankfurter et al. (1971) demonstrated the estimation error with portfolio inputs in the single index model as did Jobson and Korkie (1980, 1981). Most of us who wish to use market data to estimate our inputs are NPT people. In a normative approach, we would use the business economist's approach to understanding the phases of the business cycle and attempt to estimate portfolio inputs based on where we currently are within the business cycle. And yes, the estimation error because of infinite variance is relevant. This approach is known as top-down investing or sector rotation. An appropriate

[7] α is the utility parameter for LPM and β is the utility parameter for UPM.

investment horizon is 12–18 months because of tax efficiency and the average phase in the business cycle is around 12 months. The average business cycle is around 48 months but 10 year and 50 year cycles have also been discovered. This normative approach can also be seen in the original Barr Rosenberg bionic (fundamental) betas of 30 years ago.[8] Rosenberg would estimate an average industry beta and then make adjustments given various fundamental and economic variables. While historic data was used, there is no reason why a Bayesian approach like Mao and Särndal (1966) could not be used. Next is the practitioner-academic dissonance.

The basic conflict between academics and practitioners is that academics want to understand and explain the world around us (backward looking) and practitioners wish to forecast (forward looking). Academic explanations have to be replicable. Somebody reading an academic article can use it to replicate and achieve the same results in another part of the world. This is important because it allows us to leave a structured knowledge base that can be easily taught to future generations. In our experience, academics do not make the best practitioners and practitioners do not make the best teachers. The reason is that a practitioner evolves over time through experiential learning and generates experiential heuristics that are very difficult if not impossible to teach. Experiential heuristics deserve a great deal of respect but practitioners still have to understand what academics have explained. Both processes are laborious, take years, and are worthy of respect.

Normative portfolio theory will involve backtesting. It is also one of the ways we move academic explanation forward through our empirical studies. Unfortunately, unconstrained academic optimizers provide weird results irrelevant to practitioners even when using mean–variance. Constrained optimization moving toward linear programming (LP) heuristics and $1/n$ portfolios provide better quality portfolios for practitioners. There is in fact evidence that LP heuristics are better forecast models than any optimization model (Elton et al., 1978). Because of the business and technological cycles, portfolios have to be revised (re-estimated) and the revision period should not be longer than the current phase (contractionary phases and expansionary phases). Another strategy that may be implemented with sector rotation is rebalancing the portfolio back to its original allocations. We have known since Cheng and Deets (1971) that any kind of $1/n$ portfolio with rebalancing and low transaction costs will outperform the buy and hold strategy. The frequency of rebalancing is a negative function of transaction costs. The lower the transaction costs, the greater the frequency of rebalancing. If you are a dark pool with almost no transaction costs and you are connected directly to the exchanges' computers, you should be able to make a lot of money rebalancing. It probably was the first algorithmic trading. In fact, one of the best improvements we have made to our effective market system was negotiated transactions costs in 1975 (Nawrocki and Vaga, 2013) which increased the operational efficiency of the market. Backtesting is fine as long as the investment process being tested is adaptive. A static investment model will not work in the long run because of the dynamic disequilibrium market processes described earlier in this paper. In fact, an adaptive system does not have to forecast, just adapt to the new economic realities. This is important as it would be consistent with Nawrocki's (1984, 1995) bifurcation theory market model and Lo's (2004) adaptive market hypothesis. A complex market system that is adapting over time to changing information sets and technological change requires an adaptive investment strategy. So a long backtest over many market environments using an investment strategy that adapts to the environment should provide the best models. Next, the risk measures.

5. Necessary statistics and risk measures

Risk measures have to do two things: (1) measure the perceived risk of an investment according to your aversion to risk (and utility function), and (2) minimize the statistical error due to trying to stuff a square normal distribution risk measure into a non-normal round hole. We will still have the estimation error from using small samples. The only two risk measures approved by Markowitz as having a strong base in expected utility theory (risk-aversion) are variance and semivariance. CVaR, VaR, MVaR, Omega, Max DD (Draw Down) need not apply (and have specifically been rejected by Markowitz in his 1959 book and again by Markowitz, 2010, 2012 in recent years). All of these meet (2) but they do not meet (1). The semivariance is equivalent to LPM $n = 2$.[9] CVaR is equivalent to a conditional LPM $n = 1$ which is risk-neutral (no risk-aversion) and VaR is equivalent to LPM $n = 0$ which is the empirical CDF of the probability distribution. CVaR and LPM $n = 1$ with mean return targets are also known as a semi-mean absolute deviation which again was rejected by Markowitz as a risk measure. The pertinent question is why would you use a measure (CVaR, VaR, MVaR, Omega, Max DD) that does not measure risk so you can actually be risk-averse? These measures do not include risk-aversion as found in expected utility theory or in prospect theory. Even if you are using a Roy Safety First target return or threshold, you are only defining what level of pain you think should be avoided—you have not determined whether you are going to avoid it. This is especially true with highly skewed investment instruments that use options and other extreme leverage. VaR will give you a probability of a dollar amount but it does not tell you that the underlying distribution is severely negatively skewed and about ready to ruin your career by taking multiples of that amount from you. Any risk manager who states that it is not his/her fault because it was a 20 sigma event should receive a lifetime ban from the industry and find a new career.

If we want to integrate risk-aversion into our portfolio selection model, then we are limited to mean–variance, mean-beta, geometric mean-semivariance, and UPM/LPM. (1) If I want a portfolio model that captures any behavior

8 See Rosenberg and Guy (1976) for an example of this work.

9 At this point, we are switching from the original UPM/LPM notation of Fishburn (1977) and Holthausen (1981) used in their Figures 1 and 2 to the UPM/LPM notation used in recent articles.

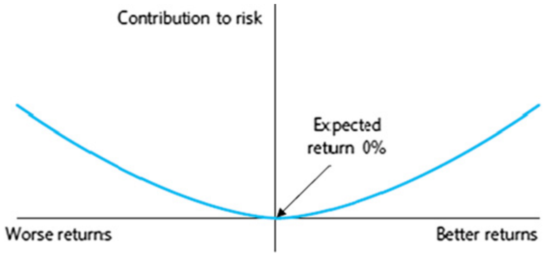

Fig. 3. Implied psychology of risk as volatility.
Source: Davies (2013).

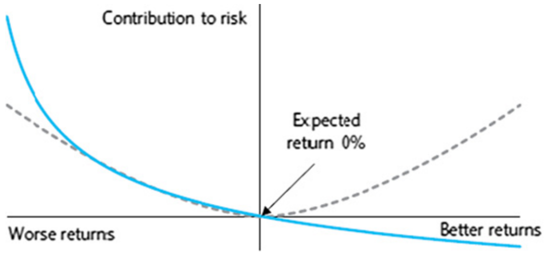

Fig. 4. Actual psychology of risk.
Source: Davies (2013).

by an investor, only the UPM/LPM model will provide it. It is consistent with Von Neumann and Morgenstern (1947), Markowitz (1952), and Kahneman and Tversky (1979), and it captures all downside risk-averse ($n > 1$), risk-neutral ($n = 1$), and risk-seeking ($0 < n < 1$) behavior by investors. (2) If I want a portfolio model that is nonparametric and will reduce statistical error without knowing the underlying distribution or using a goodness of fit test, again it is the UPM/LPM model. What makes UPMq/LPMn powerful is q and n can be any nonnegative real number other than zero, and q does not have to be equal to n. While an Omega ratio is UPM$q = 1$/LPM$n = 1$, there is no risk-aversion inherent in that calculation so the Omega ratio does not provide (1).

> "However, even when such restrictions are satisfied, or approximately satisfied, there is a contention, set forth by Domar and Musgrave (1944), Markowitz (1959), and Mao (1970a,b), among others, *that decision makers in investment contexts very frequently associate risk with failure to obtain a market return. To the extent that this contention is correct, it casts serious doubts on variance – or, for that matter, on any measure of dispersion taken with respect to a parameter (for example, mean) which changes from distribution to distribution – as a suitable measure of risk.*" Fishburn (1977).

We do see evidence of more sensible definitions of risk measures. Visually, risk contribution can be seen in Figs. 3 and 4 reproduced below from Davies (2013).

> "As before, potential outcomes that are worse than expected, add to risk, and at an increasing rate. However, outcomes that are better than expected actually *detract* from the perceived risk of the investment. Investments with potential upside thus *increase* the risk budget so real risks can be taken elsewhere in the portfolio".

These self-evident necessary adjustments require asymmetric nonlinear statistics capable of representing different risk perceptions. Also, the target need not be 0 since costs of capital will often differentiate an investor's realized gain or loss from a nominal gain or loss. In fact, a lot of utility functions break down for a zero return as demonstrated in Viole and Nawrocki (2013). Traditional statistics cannot compensate for these considerations, leading to attempts at "behavioralization".

6. Attempts to "behavioralize" statistics

6.1. Behavioral sigma

The behavioral sigma used to reconcile the risks associated with an investment is presented in Eq. (1):

$$\sigma_B^2 \approx \sigma^2 \left(1 - \frac{2\sigma}{3T}\text{skew} + \frac{\sigma^2}{3T^2}\text{kurtosis}\right) \tag{1}$$

where T is the risk-aversion parameter, "skew" is the skewness of the distribution, and excess kurtosis is denoted by "kurtosis".[10] Two concerns are immediately raised and acknowledged:

- For normal distributions, which have zero skewness or kurtosis, behavioral risk is simply equal to variance.
- The effects of these higher moments are smaller for investors with higher risk tolerance (i.e., higher values of T). Davies and de Servigny (2012)

So all other attributes equal, a positive skew will reduce σ_B^2 more for a risk-averse person (lower T) than a risk-seeking person (higher T). The question to pose however, is: Is a positive skew more or less aligned with a specific risk-aversion coefficient? If risk-seeking is concerned with above target results, it seems that a positive skew should be more aligned with that specific preference. But, a higher T in the denominator does not translate this desired risk-seeking characteristic to the new σ_B^2. It is an example of the normative prescription inherent in the formula, risk-averse investors *should* desire more positive skew relative to their risk-seeking counterparts.

Risk-aversion loves excess kurtosis (whether they *should* is a different story); yet by factoring excess kurtosis by a coefficient over T^2 has the opposite effect whereby risk-averse people (lower T) will add more risk (σ_B^2) to an investment than risk-seeking (higher T). Inversely, for negative excess kurtosis, risk-averse investors will benefit more via a lower σ_B^2. Again, this is the normative prescription whereby risk-averse investors should desire less excess kurtosis than a normal distribution.

While not meant to be an assault on the underlying principles to this metric, our criticisms reside in the inability of sigma or deviations from specific distributional characteristics to adequately convey those reasonable principles. Sigma still has to be estimated, and usually with error. Furthermore, sigma is not stationary such that the

10 See Davies and de Servigny (2012) and Davies (2013).

next period's evaluation will not equal the current estimate. While the attempt to compensate for the higher moments' effects is admirable and sensible, it is unfortunately ineffectual because expected variance is not addressed.

Furthermore, even under normality of returns, better returns *should still contribute less than losses*. As will be demonstrated later, partial moments can easily interpret given risk attitudes to desired distributional characteristics, without the above-mentioned theoretical shortfalls or conflation between normative and descriptive techniques.

6.2. Mean–variance efficient range

Statman and Clark (2013) provide a behavioral argument to extend the mean–variance efficient frontier into a range of acceptable deviations from the optimized frontier.

> "It is widely known that allocations in mean–variance optimized portfolios are sensitive to small changes in the estimates of parameters, and we know that we can never find the precise parameters. Therefore, we can never find the true mean–variance efficient frontier. But we can find an efficient range based on a range of estimates of the mean–variance parameters".
>
> "Investors' preferences for socially responsible companies might place their portfolios below optimized mean–variance efficient frontiers".
>
> "Preferences can be genuine or mere reflections of cognitive errors and misleading emotions".
>
> "Gaps between optimized portfolios produced by mean–variance optimizers and portfolios that investors prefer come from two sources. One is imprecise estimates of mean–variance parameters. The other is investor preferences beyond high expected returns and low risk."
>
> "The answer is that financial advisers must use their judgment in setting reasonable ranges and reasonable boundaries for the efficient range, recalling that judgment is inherent in mean–variance portfolio optimization."

While Shefrin and Statman (2000) and Statman and Clark (2013) represent important advances in behavioral portfolio theory, it is useful to look at other alternative behavioral portfolio models.

Generally it is the upside variance, or UPM term, responsible for the preference deviation from the efficient frontier under the classic variance = risk paradigm. These effects cannot be avoided unless the variance is parsed into "good" and "bad", per the partial moment methodology. Expanding the acceptable mean–variance range of investments may by chance incorporate the investor's desired portfolio; however, it fails to address the primitive underlying variance concerns.

6.3. Partial moments

The relationship between integration and partial moments is defined through the integral mean value theorem. The area of the function derived through both methods shares an asymptote, allowing for an empirical definition of the area. This is important in that we are no longer limited to known functions and do not have to resign ourselves to goodness of fit tests to define $f(x)$. More importantly, in social sciences where above target and below target returns have completely different meanings, partial moments do not suffer from the philosophical inconsistency whereby an above target return is used in the definition of risk. Any metric using sigma simply cannot avoid this deficiency, however modified.

Computational elegance aside, partial moments are compliant with expected utility theory and their ability to derive CDFs for any distribution allows for the integration of stochastic dominance criteria as demonstrated in Fishburn (1977) and Holthausen (1981).

Markowitz (2010) argues that mean–variance was an approximation for a wide range of risk-averse utility functions and that a more robust solution lies in the geometric mean (GM)/semideviation (S) measure.

> "For the reasons stated, I do not consider any of the above alternatives to be a satisfactory answer to the question of what type of risk measure to use in a risk-return analysis if return distributions are too spread out for functions of mean and variance to approximate expected utility well. In particular, we saw that ES_b, mean-semivariance about a return R_b, has the problem that it is linear for $R \geq b$. In this range, it does not have diminishing marginal utility of wealth. For example, its use implies indifference between receiving \$(100,000,000 + b) with certainty versus a 50–50 chance of \$$b$ or \$(200,000,000 + b). Even for less extreme cases, the lack of risk aversion among returns greater than b seems undesirable. Conversely, for returns less than b, its implied approximating utility function is the same as that of mean–variance.
>
> One cure for these problems is to combine the semideviation as a measure of risk with the geometric mean (GM) as the measure of return".

Axiomatically, every participant on the risk-aversion continuum wishes for more terminal wealth with the least amount of downside per the GM/S framework, and we note the ability of partial moments to capture these preferences. Maximizing the return components (UPM-LPM degree 1 from a zero target) will yield the same result as a geometric mean maximization since the order of the returns does not affect the calculation. Semivariance is equivalent to an LPM degree 2 from the mean target. Thus,[11]

$$\frac{GM}{S} \approx \frac{[\mathrm{UPM}(1,0,X) - \mathrm{LPM}(1,0,X)]}{LPM(2,\mu,X)}. \tag{2}$$

Incorporating the target level b we have

$$\frac{GM}{S_b} \approx \frac{[\mathrm{UPM}(1,0,X) - \mathrm{LPM}(1,0,X)]}{\mathrm{LPM}(2,b,X)}. \tag{3}$$

While Eqs. (2) and (3) are not a prescription for every investor, it speaks volumes to the ability of partial moments

[11] $\mathrm{UPM}(1,0,X)$ denotes the UPM for variable X for degree 1 and a target return of 0%. $\mathrm{LPM}(2,\mu,X)$ denotes the LPM for variable X for degree 2 and a target return of μ.

to handle the requests of the researcher. In fact, a more universal investor preference of "all of the upside with none of the downside" can be approximated with a UPM/LPM measure for individual security or a Co-UPM/Co-LPM measure for a portfolio of securities.[12]

The main advantage of using partial moments is that every investor preference from the four-fold pattern of risk described in prospect theory can be captured. Moreover, every individual investor's subjective value decision can be described in a more consistent manner than possible with traditional statistics. The framework for describing heterogeneous populations will be better served than generalizing behaviors to advance a theory.

The operational details of applying UPM-LPM portfolio theory models have been presented by Cumova and Nawrocki (2011, 2014).

7. Conclusion

Finance is and always has been behavioral. Since Bernoulli, the issue was how to operationalize the subjective interpretation of value. There have been axiomatic proposals and mathematics to support them. However, the assumptions used to bind these thoughts have come undone with the persistently heterogeneity of the population.

This paper provides an overview of the contributions of behavioral theory to the study of financial market theory, expected utility theory, and portfolio theory. The paper concludes that partial moment statistics provides the needed quantitative measures for the study of utility theory and portfolio theory in non-equilibria markets.

References

Bachelier, L., 1900. Théorie de la Spéculation. Gauthier-Villars.
Bawa, V.S., 1975. Optimal rules for ordering uncertain prospects. J. Financ. Econ. 2 (1), 95–121.
Bernoulli, D., 1954. Exposition of a new theory on the measurement of risk. Econometrica 23–36.
Boulding, K.E., 1981. Evolutionary Economics. Sage Publications, Beverly Hills.
Boulding, K.E., 1991. What is evolutionary economics? J. Evol. Econom. 1 (1), 9–17.
Cheng, P.L., Deets, M.K., 1971. Portfolio returns and the random walk theory. J. Finance 26 (1), 11–30.
Coase, R.H., 1937. The nature of the firm. Economica 4 (16), 386–405.
Cozzolino, J.M., Zahner, M.J., 1973. The maximum-entropy distribution of the future market price of a stock. Oper. Res. 21 (6), 1200–1211.
Cumova, D., Nawrocki, D., 2011. A symmetric LPM model for heuristic mean-semivariance analysis. J. Econ. Bus. 63 (2), 217–236.
Cumova, D., Nawrocki, D., 2014. Portfolio optimization in an upside potential and downside risk framework. J. Econ. Bus. 71, 68–89.
Cyert, R.M., March, J.G., 1963. A Behavioral Theory of the Firm, second ed. Cambridge, Mass., 1992.
Davies, G., 2013. What is risk (II)? Barclays Compass, July 2013. https://wealth.barclays.com/en_gb/home/research/research-centre/compass/compass-july-2013/what-is-risk.html.
Davies, G., de Servigny, A., 2012. Behavioral Investment Management: An Efficient Alternative to Modern Portfolio Theory. McGraw-Hill.
Domar, E.D., Musgrave, R.A., 1944. Proportional income taxation and risk-taking. Quart. J. Econ. 58 (3), 388–422.
Ellsberg, D., 1961. Risk, ambiguity, and the Savage axioms. Quart. J. Econ. 643–669.
Elton, E.J., Gruber, M.J., Urich, T.J., 1978. Are betas best? J. Finance 33 (5), 1375–1384.

[12] See Cumova and Nawrocki (2014) for Co-UPM/Co-LPM formulations of the portfolio problem.

Fama, E.F., 2012. Unapologetic after all these years: eugene fama defends investor rationality and market efficiency. Interview with Mark Harrison. http://blogs.cfainstitute.org/investor/2012/05/15/unapologetic-after-all-these-years-eugene-fama-defends-investor-rationality-and-market-efficiency/.
Fishburn, P.C., 1977. Mean-risk analysis with risk associated with below-target returns. Amer. Econ. Rev. 67 (2), 116–126.
Fishburn, P.C., Kochenberger, G.A., 1979. Two-piece Von Neumann–Morgenstern utility functions. Decis. Sci. 10 (4), 503–518.
Frankfurter, G.M., Phillips, H.E., 1995. Forty Years of Normative Portfolio Theory: Issues, Controversies, and Misconceptions. JAI Press.
Frankfurter, G.M., Phillips, H.E., Seagle, J.P., 1971. Portfolio selection: the effects of uncertain means, variances and covariances. J. Finan. Quant. Anal. 6 (5), 1251–1262.
Friedman, M., Savage, L.J., 1948. The utility analysis of choices involving risk. J. Polit. Econ. 56 (4), 279–304.
Georgescu-Roegen, N., 1971. The Entropy Law and the Economic Process. Harvard University Press.
Giocoli, N., 1998. The 'true' hypothesis of Daniel Bernoulli: what did the marginalists really know? Hist. Econ. Ideas 6 (2), 7–43.
Holthausen, D.M., 1981. A risk-return model with risk and return measured as deviations from a target return. Amer. Econ. Rev. 71 (1), 182–188.
Jobson, J.D., Korkie, B., 1980. Estimation for Markowitz efficient portfolios. J. Amer. Statist. Assoc. 75 (371), 544–554.
Jobson, J.D., Korkie, R.M., 1981. Putting Markowitz theory to work. J. Portfolio Manage. 7 (4), 70–74.
Kahneman, D., Tversky, A., 1979. Prospect theory: an analysis of decision under risk. Econometrica 263–291.
Lo, A.W., 2004. The adaptive markets hypothesis. J. Portfolio Manage. 30 (5), 15–29.
Mandelbrot, B.B., Hudson, R.L., 2004. The (MIS) Behaviour of Markets: A Fractal View of Risk, Ruin and Reward.
Mao, J.C., 1970a. Models of capital budgeting, EV vs. ES. J. Finan. Quant. Anal. 657–675.
Mao, J.C., 1970b. Essentials of portfolio diversification strategy. J. Finance 25 (5), 1109–1121.
Mao, J.C., Särndal, C.E., 1966. A decision theory approach to portfolio selection. Manag. Sci. 12 (8), B-323.
March, J.G., Simon, H.A., 1958. Organizations.
Markowitz, H., 1952. The utility of wealth. J. Polit. Econ. 60 (2), 151–158.
Markowitz, H., 1959. Portfolio Selection: Efficient Diversification of Investments, Vol. 16. Yale University press.
Markowitz, H., 2010. Portfolio theory: as I still see it. Annu. Rev. Financ. Econ. 2, 1–23.
Markowitz, H., 2012. The omega ratio. Working Paper.
Murphy, R.E., 1965. Adaptive Processes in Economic Systems, Vol. 20. Academic Press, New York.
Nawrocki, D., 1984. Entropy, bifurcation and dynamic market disequilibrium. Financ. Rev. 19 (2), 266–284.
Nawrocki, D.N., 1995. Expectations, technological change, information and the theory of financial markets. Int. Rev. Financ. Anal. 4 (2), 85–105.
Nawrocki, D., Vaga, T., 2013. A bifurcation model of market returns. Quant. Finance 1–20 (ahead-of-print).
Nicolis, G., Prigogine, I., 1977. Self-Organization in Nonequilibrium Systems.
Peters, E.E., 1991. Chaos and Order in the Capital Markets. A New View of Cycle, Prices and Market Volatility. J. Wiley.
Peters, E.E., 1994. Fractal Market Analysis: Applying Chaos Theory to Investment and Economics, Vol. 24. Wiley, New York.
Roll, R., 1977. A critique of the asset pricing theory's tests part I: on past and potential testability of the theory. J. Financ. Econ. 4 (2), 129–176.
Rosenberg, B., Guy, J., 1976. Prediction of beta from investment fundamentals: part one, prediction criteria. Financ. Anal. J. 60–72.
Shannon, C.E., Weaver, W., 1949. The Mathematical Theory of Communication. University of Illinois Press, Urbana, IL.
Shefrin, H., Statman, M., 2000. Behavioral portfolio theory. J. Finan. Quant. Anal. 35 (02), 127–151.
Statman, M., Clark, J., 2013. End the Charade: replacing the efficient frontier with the efficient range. Available at SSRN: http://ssrn.com/abstract=2230548.
Viole, F., Nawrocki, D., 2011. The utility of wealth in an upper-and lower-partial moment fabric. J. Investing 20 (2), 58–85.
Viole, F., Nawrocki, D., 2013. An analysis of heterogeneous utility benchmarks in a zero return environment. Int. Rev. Financ. Anal. 190–198.
Von Neumann, J., Morgenstern, O., 1947. The Theory of Games and Economic Behavior.
Wiener, N., 1948. Cybernetics. Hermann, Paris.
Williamson, O.E., 2002. The theory of the firm as governance structure: from choice to contract. J. Econ. Perspect. 16 (3), 171–195.